Questions

Q1.

The curve *C* has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point *P* (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for question = 4 marks)

Q2.

A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of *x* for which the curve is increasing.

(2)

(Total for question = 5 marks)

Q3.

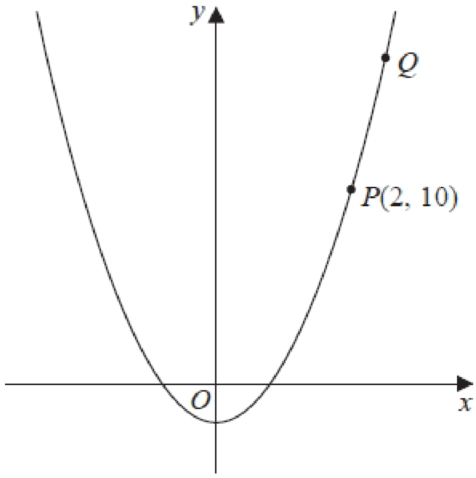

Figure 1

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line PQ, giving your answer in terms of h in simplest form.

(3)

(c) Explain briefly the relationship between part (b) and the answer to part (a).

(1)

(Total for question = 6 marks)

Q4.

The curve C has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is –3

(a) (i) show that the value of a is –2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

(2)

(Total for question = 11 marks)

Q5.

A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

(Total for question = 7 marks)

Q6.

The curve *C*, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve *C* passes through the origin *O*

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for sin 2*y* to find an equation linking *x* and *y* for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (*x*, *y*) lying on *C*,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where *a* and *b* are constants to be found.

(3)

(Total for question = 7 marks)

Q7.

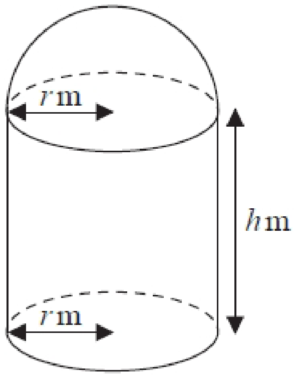

Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is 6 m^3.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

(Total for question = 10 marks)

Q8.

The curve *C* has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that *C* has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

(Total for question = 7 marks)

Q9.

The curve C has equation

$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$

(a) Find

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

(Total for question = 7 marks)

Q10.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

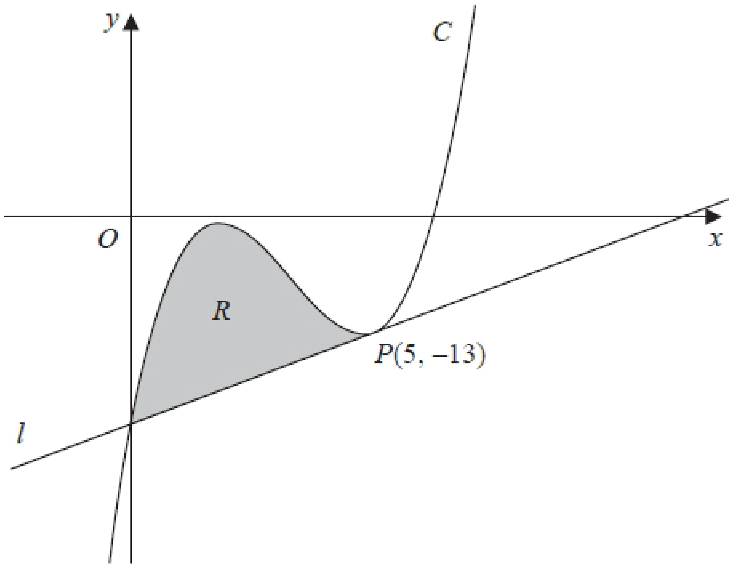

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

(Total for question = 9 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Attempt to differentiate	M1	1.1a
	$\frac{dy}{dx} = 4x - 12$	A1	1.1b
	Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = ...$	M1	1.1b
	$\Rightarrow \frac{dy}{dx} = 8$	A1ft	1.1b
			(4 marks)

Notes

M1 : Differentiation implied by one correct term
A1 : Correct differentiation
M1 : Attempts to substitute $x = 5$ into their derived function
A1ft: Substitutes $x = 5$ into **their** derived function **correctly** i.e. Correct calculation of their $f'(5)$ so follow through slips in differentiation

Q2.

Question	Scheme	Marks	AOs
(a)	$x^n \to x^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right) = 6x - \frac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
(b)	Attempts $6x - \frac{24}{x^2} > 0 \Rightarrow x >$	M1	1.1b
	$x > \sqrt[3]{4}$ or $x \ldots \sqrt[3]{4}$	A1	2.5
		(2)	
		(5 marks)	

Notes

(a)

M1: $x^n \to x^{n-1}$ for any correct index of x. Score for $x^2 \to x$ or $x^{-1} \to x^{-2}$

Allow for unprocessed indices. $x^2 \to x^{2-1}$ oe

A1: Sight of either $6x$ or $-\frac{24}{x^2}$ which may be un simplified.

Condone an additional term e.g. + 2 for this mark

The indices now must have been processed

A1: $\frac{dy}{dx} = 6x - \frac{24}{x^2}$ or exact simplified equivalent. Eg accept $\frac{dy}{dx} = 6x^1 - 24x^{-2}$

You do not need to see the $\frac{dy}{dx}$ and you should isw after a correct simplified answer.

(b)

M1: Sets an allowable $\frac{dy}{dx} \ldots 0$ and proceeds to $x\ldots$ via an allowable intermediate equation

$\frac{dy}{dx}$ must be in the form $Ax + Bx^{-2}$ where $A, B \neq 0$

and the intermediate equation must be of the form $x^p \ldots q$ oe

Do not be concerned by either the processing, an equality or a different inequality.

It may be implied by $x =$ awrt 1.59

A1: $x > \sqrt[3]{4}$ or $x \geq \sqrt[3]{4}$ oe such as $x > 4^{\frac{1}{3}}$ or $x \geq 2^{\frac{2}{3}}$

Q3.

Question	Scheme	Marks	AOs
(a)	Attempts to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x \Rightarrow$ gradient of tangent at P is 12	A1	1.1b
		(2)	
(b)	Gradient $PQ = \frac{3(2+h)^2 - 2 - 10}{(2+h)-2}$ oe	B1	1.1b
	$= \frac{3(2+h)^2 - 12}{(2+h)-2} = \frac{12h + 3h^2}{h}$	M1	1.1b
	$= 12 + 3h$	A1	2.1
		(3)	
(c)	Explains that as $h \to 0,\ 12 + 3h \to 12$ and states that the gradient of the chord tends to the gradient of (the tangent to) the curve	B1	2.4
		(1)	
			(6 marks)

Notes

(a)

M1: Attempts to differentiate, allow $3x^2 - 2 \to \ldots x$ and substitutes $x = 2$ into their answer

A1: cso $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x \Rightarrow$ gradient of tangent at P is 12

(b)

B1: Correct expression for the gradient of the chord seen or implied.

M1: Attempts $\frac{\delta y}{\delta x}$, condoning slips, and attempts to simplify the numerator. The denominator must be h

A1: cso $12 + 3h$

(c)

B1: Explains that as $h \to 0,\ 12 + 3h \to 12$ and states that the gradient of the chord tends to the gradient of the curve

Q4.

Question	Scheme	Marks	AOs
(a) (i)	Uses $\frac{dy}{dx} = -3$ at $x = 2 \Rightarrow\ 12a + 60 - 39 = -3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a + 60 - 39 = -3 \Rightarrow 12a = -24 \Rightarrow a = -2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $\quad 10 = 8a + 60 - 78 + b$	M1	3.1a
	Subs $a = -2$ into $10 = 8a + 60 - 78 + b \Rightarrow b = 44$	A1	1.1b
		(4)	
(b)	$f(x) = -2x^3 + 15x^2 - 39x + 44 \Rightarrow f'(x) = -6x^2 + 30x - 39$	B1	1.1b
	Attempts to show that $-6x^2 + 30x - 39$ has no roots Eg. calculates $b^2 - 4ac = 30^2 - 4\times -6\times -39 = -36$	M1	3.1a
	States that as $f'(x) \neq 0 \Rightarrow$ hence $f(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3 + 15x^2 - 39x + 44 \equiv (x-4)(-2x^2 + 7x - 11)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	
			(11 marks)

Notes

(a)(i)

M1: Attempts to use $\frac{dy}{dx} = -3$ at $x = 2$ to form an equation in a. Condone slips but expect to see two of the powers reduced correctly

A1*: Correct differentiation with one correct intermediate step before $a = -2$

(a)(ii)

M1: Attempts to use the fact that $(2,10)$ lies on C by setting up an equation in a and b with $a = -2$ leading to $b = ...$

A1: $b = 44$

(b)

B1: $f'(x) = -6x^2 + 30x - 39$ oe

M1: Correct attempt to show that "$-6x^2 + 30x - 39$" has no roots.
This could involve an attempt at

- finding the numerical value of $b^2 - 4ac$
- finding the roots of $-6x^2 + 30x - 39$ using the quadratic formula (or their calculator)
- completing the square for $-6x^2 + 30x - 39$

A1*: A fully correct method with reason and conclusion. Eg as $b^2 - 4ac = -36 < 0, f'(x) \neq 0$ meaning that no stationary points exist

(c)

M1: For an attempt at division (seen or implied) Eg $-2x^3 + 15x^2 - 39x + b \equiv (x-4)\left(-2x^2 ... \pm \frac{b}{4}\right)$

A1: $(x-4)(-2x^2 + 7x - 11)$ Sight of the quadratic with no incorrect working seen can score both marks.

(d)

See scheme. You can follow through on their value for b

Q5.

Question	Scheme	Marks	AOs
(a)	(i) $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x = 4$ into their $\frac{dy}{dx} = 2\times4 - 2 - 12\times4^{-\frac{1}{2}} = ...$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x = 4$ into their $\frac{d^2y}{dx^2} = 2 + 6\times4^{-\frac{3}{2}} = (2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2} = 2.75 > 0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

(a)(i)

M1: Differentiates to $\frac{dy}{dx} = Ax + B + Cx^{-\frac{1}{2}}$ **A1:** $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}$ (Their $\frac{dy}{dx}$ must have a negative or fractional index)

(b)

M1: Substitutes $x = 4$ into their $\frac{dy}{dx}$ and attempts to evaluate. There must be evidence $\left.\frac{dy}{dx}\right|_{x=4} = ...$

Alternatively substitutes $x = 4$ into an equation resulting from $\frac{dy}{dx} = 0$ Eg. $\frac{36}{x} = (x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓, QED for a minimal conclusion

Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe

Alt Shows that $x = 4$ is a root of the resulting equation and states "hence there is a stationary point"
All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x = 4$ into their $\frac{d^2y}{dx^2}$ and calculates its value, or implies its sign by a statement such as when $x = 4 \Rightarrow \frac{d^2y}{dx^2} > 0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x = 4$ or calculates the value of y either side of $x = 4$.

A1ft: For **a correct calculation, a valid reason and a correct conclusion**. Ignore additional work where candidate finds $\frac{d^2y}{dx^2}$ left and right of $x = 4$. Follow through on an incorrect $\frac{d^2y}{dx^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".

Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

Q6.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	This mark is given for differentiating and inverting
	At (0, 0), $\frac{dy}{dx} = \frac{1}{8}$	A1	This mark is given for finding $\frac{dy}{dx}$ when $y = 0$
(b)(i)	$\sin 2y \approx 2y \Rightarrow x \approx 8y$	B1	This mark is given for finding an approximation for x
(b)(ii)	When x and y are small, $x = 4\sin 2y$ approximates to the line $x = 8y$	B1	This mark is given for a valid explanation of the relationship between x and y when both are small
(c)	$\sin^2 2y + \cos^2 2y = 1$ $\Rightarrow \cos^2 2y = 1 - \sin^2 2y$ $x = 4\sin 2y \Rightarrow \sin^2 2y = \left(\frac{x}{4}\right)^2$	M1	This mark is given for a method to use find an expression for $\sin^2 2y$ in terms of x
	$\frac{dy}{dx} = \frac{1}{8\cos 2y} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$	A1	This mark is given for an unsimplified expression for $\frac{dy}{dx}$
	$\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	This mark is given for a fully correct answer with $a = 2$ and $b = 16$

Q7.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	This mark is given for a method to find the volume of the cylinder and the semi-hemisphere
	$A = 3\pi r^2 + 2\pi\left(\frac{6 - \frac{2}{3}\pi r^3}{\pi r}\right)$	M1	This mark is given for a method to find the surface area of the tank
		A1	This mark is given for finding an expression for the surface area of the tank
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4\pi r^2}{3} = \frac{12}{r} + \frac{5\pi r^2}{3}$	A1	This mark is given for a fully correct proof to show the surface area of the tank as required
(b)	$A = \frac{12}{r} + \frac{5\pi r^2}{3} \Rightarrow \frac{dA}{dr} = -\frac{12}{r^2} + \frac{10\pi r}{3}$	M1	This mark is given for a method to differentiate to find r
		A1	This mark is given for accurately differentiating to find r
	When $\frac{dA}{dr} = 0$, $-\frac{12}{r^2} + \frac{10\pi r}{3} = 0$ $r^3 = \frac{18}{5\pi}$	M1	This mark is given for a method to set $\frac{dA}{dr} = 0$ to find a value for r
	$r = 1.046$	A1	This mark is given for finding the radius for which the surface area is a minimum
(c)	$A = \frac{12}{1.046} + \frac{5\pi(1.046)^2}{3}$	M1	This mark is given for a method to substitute a value for r
	$A = 17\ \text{m}^2$	A1	This mark is given for correctly finding the minimum surface area of the tank (to the nearest integer)
			(Total 10 marks)

Q8.

Question	Scheme	Marks	AOs
(a)	(i) $\frac{dy}{dx} = 12x^3 - 24x^2$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x = 2$ into their $\frac{dy}{dx} = 12\times 2^3 - 24\times 2^2$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x = 2$ into their $\frac{d^2y}{dx^2} = 36\times 2^2 - 48\times 2$	M1	1.1b
	$\frac{d^2y}{dx^2} = 48 > 0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
		(7 marks)	

Notes:
(a)(i) **M1:** Differentiates to a cubic form **A1:** $\frac{dy}{dx} = 12x^3 - 24x^2$ **(a)(ii)** **A1ft:** Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 36x^2 - 48x$
(b) **M1:** Substitutes $x = 2$ into their $\frac{dy}{dx}$ **A1:** Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" All aspects of the proof must be correct
(c) **M1:** Substitutes $x = 2$ into their $\frac{d^2y}{dx^2}$ Alternatively calculates the gradient of C either side of $x = 2$ **A1ft:** For a correct calculation, a valid reason and a correct conclusion. Follow through on an incorrect $\frac{d^2y}{dx^2}$

Q9.

Question	Scheme	Marks	AOs
(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	
(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = \ldots$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = \ldots \quad \left(\frac{d^2y}{dx^2}\right)_{x=1.2} = \ldots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0, \quad \left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = \ldots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = \ldots \quad \left(\frac{dy}{dx}\right)_{x=1.2} = \ldots$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0, \quad \left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
			(7 marks)

(a)(i)
M1: $x^n \to x^{n-1}$ for at least one power of x

A1: $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(b)(i)

M1: Substitutes $x = 1$ into their $\frac{dy}{dx}$

A1: Obtains $\frac{dy}{dx} = 0$ following a correct derivative and makes a conclusion which can be minimal e.g. tick, QED etc. which may be in a preamble e.g. stationary point when $\frac{dy}{dx} = 0$ and then shows $\frac{dy}{dx} = 0$

Alternative:

M1: Attempts to solve $\frac{dy}{dx} = 0$ by factorisation. This may be by using the factor of $(x - 1)$ or possibly using a calculator to find the roots and showing the factorisation. Note that they may divide by 4 before factorising which is acceptable. Need to either see either $4(x-1)^2(5x-8)$ or $(x-1)^2(5x-8)$ for the factorisation or $x = \frac{8}{5}$ and $x = 1$ seen as the roots.

A1: Obtains $x = 1$ and makes a conclusion as above

(b)(ii)
M1: Considers the value of the second derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.
(NB $\frac{d^2y}{dx^2} = (x-1)(60x-84)$ so may use this factorised form when considering $x < 1$, $x > 1$ for sign change of second derivative)
A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that the stationary point is a point of inflection. Sufficient reason is e.g. "sign change"/ "$> 0, < 0$". If values are given they should be correct (but be generous with accuracy) but also just allow "> 0" and "< 0" provided they are correctly paired. The interval must be where $x < 1.4$

Alternative 1 for (b)(ii)
M1: Shows that second derivative at $x = 1$ is zero and **then finds the third derivative at $x = 1$**
A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is "$\neq 0$" but must follow a correct third derivative and a correct value if evaluated. For reference $\left(\frac{d^3y}{dx^3}\right)_{x=1} = -24$

Alternative 2 for (b)(ii)
M1: Considers the value of the first derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.
A1: Fully correct work with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is e.g. "same sign"/"both negative"/"$< 0, < 0$". If values are given they should be correct (but be generous with accuracy). The interval must be where $x < 1.4$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
f'(x)	-32	-24.3	-17.92	-12.74	-8.64	-5.5	-3.2	-1.62	-0.64	-0.14	0
f''(x)	84	70.2	57.6	46.2	36	27	19.2	12.6	7.2	3	0

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7
f'(x)	-0.1	-0.32	-0.54	-0.64	-0.5	0	0.98
f''(x)	-1.8	-2.4	-1.8	0	3	7.2	12.6

Q10.

Question	Scheme	Marks	AOs
(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \frac{dy}{dx} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5} = 3 \times 5^2 - 20 \times 5 + 27 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 5)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
		(4)	
(b)	Both *C* and *l* pass through (0, −23) and so *C* meets *l* again on the *y*-axis	B1	2.2a
		(1)	
(c)	$\pm \int \left(x^3 - 10x^2 + 27x - 23 - (2x - 23)\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right]_0^5$ $= \left(\frac{625}{4} - \frac{1250}{3} + \frac{625}{2}\right)(-0)$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative 1:		
	$\pm \int \left(x^3 - 10x^2 + 27x - 23\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right]_0^5 + \frac{1}{2} \times 5(23 + 13)$ $= -\frac{455}{12} + 90$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
	(c) Alternative 2:		
	$\int \left(x^3 - 10x^2 + 27x\right) dx = \left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2\right]_0^5 - \frac{1}{2} \times 5 \times 10$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
			(9 marks)

Notes

(a)
B1: Correct derivative
M1: Substitutes $x = 5$ into their derivative. This may be implied by their value for $\frac{dy}{dx}$
M1: Fully correct straight line method using $(5, -13)$ and their $\frac{dy}{dx}$ at $x = 5$
A1: cao. Must see the full equation in the required form.
(b)
B1: Makes a suitable deduction.

Alternative via equating l and C and factorising e.g.

$$x^3 - 10x^2 + 27x - 23 = 2x - 23$$
$$x^3 - 10x^2 + 25x = 0$$
$$x\left(x^2 - 10x + 25\right) = 0 \Rightarrow x = 0$$

So they meet on the y-axis

(c)
M1: For an attempt to integrate $x^n \to x^{n+1}$ for $\pm$" $C - l$"
A1ft: Correct integration in any form which may be simplified or unsimplified. (follow through their equation from (a))

If they attempt as 2 separate integrals e.g. $\int \left(x^3 - 10x^2 + 27x - 23\right)\, dx - \int (2x - 23)\, dx$ then award this mark for the correct integration of the curve as in the alternative.

If they combine the curve with the line first then the subsequent integration must be correct or a correct ft for their line and allow for $\pm$" $C - l$"

dM1: Fully correct strategy for the area. Award for use of 5 as the limit and condone the omission of the "– 0". **Depends on the first method mark.**
A1: Correct exact value

Alternative 1:
M1: For an attempt to integrate $x^n \to x^{n+1}$ for $\pm C$
A1: Correct integration for $\pm C$
dM1: Fully correct strategy for the area e.g. correctly attempts the area of the trapezium and subtracts the area enclosed between the curve and the x-axis. Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the trapezium **and** the subtraction.

May see the trapezium area attempted as $\int (2x - 23)\, dx$ in which case the integration and use of the limits needs to be correct or correct follow through for their straight line equation.

Depends on the first method mark.

A1: Correct exact value

Note if they do $l - C$ rather than $C - l$ and the working is otherwise correct allow full marks if their final answer is given as a positive value. E.g. correct work with $l - C$ leading to $-\frac{625}{12}$ and then e.g. hence area is $\frac{625}{12}$ is acceptable for full marks.

If the answer is left as $-\frac{625}{12}$ then score A0

Alternative 2:

M1: For an attempt to integrate $x^n \to x^{n+1}$ for $(C + 23)$

A1: Correct integration for $(C + 23)$

dM1: Fully correct strategy for the area e.g. correctly attempts the area of the triangle and subtracts from the area under the curve

Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the triangle **and** the subtraction.

Depends on the first method mark.